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Heat transfer enhancement in oscillatory flows of Newtonian and viscoelastic fluids

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ABSTRACT

The work by U.H. Kurzweg for the enhanced longitudinal heat transfer of a Newtonian fluid in zero-mean oscillatory laminar flows in tubes subjected to an axial temperature gradient [U.H. Kurzweg, J. Heat Transfer 107 (1985) 459–462] is generalized for the case of a viscoelastic Maxwell fluid. While Kurzweg discovered that a Newtonian fluid exhibits a single maximum value of the effective diffusivity for a specific oscillation frequency, several maxima for different resonant frequencies are found in the case of the Maxwell fluid. The absolute maximum of the enhanced thermal diffusivity for the viscoelastic fluid and, consequently, the axial heat transfer in the tube, may be much higher than those for the Newtonian fluid. Since this absolute maximum increases as the radius of the tube decreases, a possible application may be to improve the efficiency of micro- and nano-thermal devices through the enhancement of the heat transfer rates in those devices. We provide two specific examples of heat transfer enhancement: a standard viscoelastic fluid (CPyCl/NaSal) oscillating in a macroscopic tube (scale of centimeters) and water oscillating at high frequencies in a tube of nanometric scale under conditions similar to those used experimentally in water nanoresonators.

1. Introduction

Heat transfer enhancement in fluids plays an important role in the design of many traditional engineering devices such as heat exchangers and cooling modules (see for instance [1] and references therein). Recently, it has also become a must in processes involving heat removal from components such as electronic chips and other similar high energy devices, as well as in nanofluidic applications [2,3].

Among different enhancement methods, the use of oscillatory flows deserves a special mention. In fact, it has been determined that the existence of an oscillatory flow may improve a given transport process. For instance, the axial dispersion of contaminants within laminar oscillatory flows in capillary tubes is considerably larger than that obtained by pure molecular diffusion in the absence of flow [4,5]. Moreover, it has been found that the dynamic permeability of a viscoelastic fluid flowing in a tube can be substantially enhanced at specific resonant oscillation frequencies [6–8]. Under certain conditions, an enhanced flow rate can be achieved. Recently, this resonant behavior was experimentally observed and the enhancement at the frequencies predicted by the theory was confirmed [9,10]. Owing to the analogy between heat and mass transfer, it was recognized by Kurzweg [11–13] that in a zero-mean oscillatory flow of a Newtonian fluid in a duct connecting two fluid reservoirs at different temperatures, the effective thermal diffusivity reaches a maximum for a specific oscillation frequency. This leads to an enhanced longitudinal heat transfer which involves no net mass transfer as long as the flow remains laminar. In the presence of a longitudinal temperature gradient, the enhancement is produced by the combination of two mechanisms of thermal energy transport, namely, the lateral diffusive transport through boundary layers and walls and the periodic longitudinal convective transport [13,14]. This can result in a very significant increase in the longitudinal heat transport capability of the fluid once tuned conditions are reached. In fact, using water in a high-frequency oscillatory flow within a capillary bundle connecting two reservoirs at different temperatures, Kurzweg and Zhao [11] found that effective thermal diffusivities are about four orders of magnitude larger than the molecular diffusivity of water, the heat transfer rates being comparable to those achieved with heat pipes.

Originally, the work of Kurzweg pointed to applications such as the removal of heat from radioactive fluids or from hazardous chemical solutions where a rapid removal of heat without an accompanying convective mass transfer is required [15]. Nowadays, Kurzweg's method for heat transfer enhancement using oscillatory flows becomes potentially interesting for micro and nanofluidic applications. In fact, the important characteristics of

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Nomenclature

a	pipe radius (m)
A	cross-sectional area of the tube (m^2)
c	specific heat ($J\,kg^{-1}\,K^{-1}$)
De	Deborah number ($=t_m\eta/a^2\rho$)
H_x	ratio of heat fluxes
J_0	cylindrical Bessel function of zeroth order
J_1	cylindrical Bessel function of first order
k	fluid thermal conductivity ($W\,m^{-1}\,K^{-1}$)
L	characteristic length, pipe length (m)
p	pressure ($N\,m^{-2}$)
Pr	Prandtl number ($=\eta/\rho\alpha$)
Pe	Péclet number ($=v_o a/\alpha$)
q''_m	molecular heat flux
q''_o	heat flux under oscillatory flow
r	radial coordinate (m)
t	time (s)
t_m	fluid relaxation time (s)
T	fluid temperature (K)
$\mathbf{v}$	fluid velocity vector ($m\,s^{-1}$)
v	axial fluid velocity component ($m\,s^{-1}$)
W_i	Weissenberg number ($=\omega t_m$)
Wo	Womersley number $= a\sqrt{\rho\omega/\eta}$
x	axial coordinate (m)

Greek symbols

α	fluid thermal diffusivity ($m^2\,s^{-1}$)
α_e	effective fluid thermal diffusivity ($m^2\,s^{-1}$)
$\hat{\alpha}_e$	dimensionless effective thermal diffusivity ($=\alpha_e/\alpha$)
β_v	frequency parameter of the Bessel differential equation for the velocity
β_T	frequency parameter of the Bessel differential equation for the temperature
γ	time-averaged axial temperature gradient ($K\,m^{-1}$)
η	dynamic viscosity ($kg\,m^{-1}\,s^{-1}$)
ρ	mass density ($kg\,m^{-3}$)
$\tilde{\tau}$	viscous stress tensor ($kg\,m^{-1}\,s^{-2}$)
ω	angular frequency (radians s^{-1})

this heat transfer enhancement process are retained provided that the flow is strictly laminar and, consequently, it is applicable for low-Reynolds number flows. Kurzweg [13] stated that optimum heat-transfer devices based on this concept require narrow channels in which viscous effects are large enough to prevent the appearance of turbulence. Moreover, the process is less efficient at very low frequencies so that middle- and high-frequency applications should be preferred. On the other hand, in a recent paper Yakhot and Colosqui [16] analyzed a flow generated by an infinite plate in oscillatory motion (Stokes' second problem) in the whole range of oscillating dimensionless frequencies $0 \leqslant \omega t_m \leqslant \infty$, where ω is the angular oscillation frequency and t_m is the Maxwell relaxation time. Their solution describes a transition, observed experimentally, from a viscoelastic behavior of a Newtonian fluid ($\omega t_m \ll 1$) to a dynamics dominated by pure elastic effects ($\omega t_m \gg 1$). They showed that results agree with experiments on nanoresonators operating in a wide range of pressure and frequency variation in both gases and water [17]. An important remark of this paper is that high-frequency low-Reynolds number flows, where the inertial contributions are negligibly small, are rheological. More recently Ekinci et al. [18], have emphasized that many interesting phenomena, including enhanced heat transfer in nanoparticle-seeded fluids, occur in a range of parameters where the Newtonian fluid approximation breaks down. In this context, the exploration of oscillating viscoelastic fluids for heat transfer enhancement purposes becomes relevant.

A number of heat transfer studies in confined viscoelastic fluids have been recently conducted [19–21]. However, the use of oscillating viscoelastic fluids for the enhanced transport of heat, apart from a preliminary exploration for solar applications [22] has, to our knowledge, not been considered so far. This is precisely the main aim of this paper. Here, the analysis performed by Kurzweg [12] for the enhanced longitudinal heat transfer in a zero-mean oscillatory laminar flow in a tube connecting two reservoirs at different temperatures, is generalized in two ways. First, we provide an analytical solution for the fluid temperature in a cylindrical duct under insulating wall conditions that is applicable to both Newtonian and Maxwellian fluids in oscillatory motion and, most likely with minor modifications, also for a variety of other viscoelastic fluids because of the linear suggested regime. We note that, in contrast with this analytical result, for the case of a Newtonian fluid Kurzweg [12] provided an approximate solution based on a multiscale expansion technique that is only valid for small values of the product Wo^2Pr, where $Wo = a\sqrt{\rho\omega/\eta}$ and $Pr = \eta/\rho\alpha$ are the Womersley and Prandtl numbers, respectively. Here, ρ, η and $\alpha = k/(\rho c)$ are the mass density, dynamic viscosity and molecular thermal diffusivity with c and k being the specific heat and thermal conductivity of the fluid, respectively, while a is the characteristic length scale. Second, using the fluid velocity and temperature analytic profiles, we calculate an explicit expression of the effective thermal diffusivity for a Maxwell fluid and explore its behavior in a wide range of Womersley and Prandtl numbers, including the Newtonian limit where Kurzweg's results are recovered [13]. The analysis of the effective thermal diffusivity enables us to demonstrate the longitudinal heat transfer enhancement for viscoelastic fluids in oscillatory motion and the existence of multiple resonant frequencies.

The paper is organized as follows. In Section 2 we set out the theoretical framework and compute the velocity and temperature fields for both the Newtonian and the Maxwell fluid. This is followed in Section 3 by the computation of the enhanced thermal diffusivity and the dimensionless heat flux. The paper is closed in Section 4 with some discussion and concluding remarks.

2. Theoretical model

We consider an oscillating incompressible laminar flow in a tube of radius a and length L. The oscillatory motion of the working fluid is driven by a harmonic pressure gradient applied in the longitudinal x-direction. We assume that the ends of the tube are connected to thermal reservoirs of constant but different temperature, that is, $T(x = 0) = T_1$ and $T(x = L) = T_2$, where $T_1 > T_2$. Moreover, the tube wall is assumed to be thermally insulated so that heat can only be transferred in the x-direction.

We start the analysis with the governing balance equations, namely, the continuity equation for an incompressible fluid,

$$\nabla \cdot \mathbf{v} = 0, \tag{1}$$

the momentum balance equation,

$$\rho \frac{\partial \mathbf{v}}{\partial t} + \rho(\mathbf{v} \cdot \nabla)\mathbf{v} = -\nabla p - \nabla \cdot \tilde{\tau}, \tag{2}$$

and the energy balance equation,

$$\rho c\left(\frac{\partial T}{\partial t}+\mathbf{v}\cdot\nabla T\right)=k\nabla^2 T. \qquad (3)$$

Here $\mathbf{v}$, p and T are velocity, pressure and temperature of the fluid, respectively, while $\tilde{\tau}$ is the viscous stress tensor. In Eq. (3) the viscous heating term has been neglected since usually this term is only important when dealing with high Prandtl number fluids, as viscous oils. In fact, typical temperature differences produced by viscous dissipation are $\Delta T = Pr(\omega\Delta x)^2/c$ [13]. Therefore, viscous heating will be important provided that ΔT compares with the temperature difference established by the fluid reservoirs at the extreme of the tube.

We consider that $\tilde{\tau}$ satisfies the linear form of the Maxwell model, namely

$$t_m\frac{\partial\tilde{\tau}}{\partial t}=-\eta\nabla\mathbf{v}-\tilde{\tau}. \qquad (4)$$

It is clear that in the limit $t_m \to 0$, Eq. (4) reduces to the constitutive equation for a Newtonian fluid. Therefore, in what follows we address the problem in a general form for a linear Maxwell fluid bearing in mind that the Newtonian case can be recovered by taking the limit $t_m \to 0$. Recent experimental results indicate that the linear approximation (4) is suitable for a reliable description for low Reynolds numbers ($<10^{-3}$) and low ($0.6\ \mathrm{s}^{-1}$) shear rates [9,10]. This is precisely the flow regime that will be considered in this paper.

2.1. Velocity profile

We first consider the oscillatory laminar flow of a Maxwell fluid in a long tube of circular cross-section. Border effects at the ends of the tube can be disregarded for distances from the edge larger than the entrance length which is proportional to the square of the thickness of the oscillating boundary layer on the walls of the tube [23]. Therefore, assuming that the flow is fully developed, the only velocity component is in the axial direction and takes the form $v = v(r,t)$, so that the continuity equation is identically satisfied. Under these conditions, Eqs. (2) and (4) can be combined to give the equation of motion in the form

$$t_m\frac{\partial^2 v}{\partial t^2}+\frac{\partial v}{\partial t}=-\frac{1}{\rho}\left[t_m\frac{\partial}{\partial t}\left(\frac{\partial p}{\partial x}\right)+\frac{\partial p}{\partial x}\right]+\frac{\eta}{\rho}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v}{\partial r}\right). \qquad (5)$$

By requiring axial symmetry of the velocity profile and the non-slip condition at the wall, the boundary conditions that must be satisfied by Eq. (5) are

$$\frac{\partial v}{\partial r}(0,t)=0, \qquad (6)$$

$$v(a,t)=0. \qquad (7)$$

We consider that the zero-mean oscillatory flow is produced by a harmonic pressure gradient that can be expressed as the real part of $\partial p/\partial x = P_x e^{-i\omega t}$, where P_x is the constant amplitude of the pressure gradient. The axial velocity component can then be expressed as the real part of $v(r,t) = V(r)e^{-i\omega t}$. Therefore, from Eq. (5), the function $V(r)$ satisfies the equation

$$\frac{d^2V}{dr^2}+\frac{1}{r}\frac{dV}{dr}+\frac{(t_m\omega^2-i\omega)}{\eta/\rho}V=\frac{(1+i\omega t_m)}{\eta}P_x. \qquad (8)$$

The solution of Eq. (8) that is compatible with boundary conditions (6) and (7) is given by

$$V(r)=\Phi(\omega)\left[1-\frac{J_0(\beta_v r)}{J_0(\beta_v a)}\right]P_x, \qquad (9)$$

where

$$\Phi(\omega)=\frac{1+i\omega t_m}{\beta_v^2\eta}=i\frac{a^2 De}{\eta\omega t_m}=i\frac{a^2}{\eta Wo^2}, \qquad (10)$$

and

$$\beta_v^2(\omega)=\frac{1}{a^2 De}\left[(\omega t_m)^2-i\omega t_m\right]=\frac{Wo^2}{a^2}\left[De\,Wo^2-i\right]. \qquad (11)$$

Here $De = \eta t_m/a^2\rho$ is the Deborah number that gives the ratio of the relaxation time t_m to the viscous diffusion time, $\rho a^2/\eta$. Likewise, the square of the Womersley number, $Wo^2 = a^2\rho\omega/\eta$, is the ratio of the viscous diffusion time to the characteristic *oscillation* time, $1/\omega$. In addition, J_0 is the cylindrical Bessel function of the first kind and zeroth order. It is worth mentioning that the product $De\,Wo^2$ gives the Weissenberg number $W_i = \omega t_m$, whose value is sometimes used to distinguish between the Newtonian-like or the elastic-like character of the system [18]. However, we prefer to stick here to the Deborah number for this purpose since the Newtonian limit is obtained directly when $t_m \to 0$.[1] As a matter of fact, the velocity profile (Eqs. (9)–(11)) reduces to the one of the oscillating Newtonian flow through a pipe in the limit $De \to 0$ [12,24].

For a proper comparison with Kurzweg's results, it is convenient to introduce the tidal displacement, Δx, that represents the cross-stream averaged maximum axial distance which the fluid elements travel during one half period of the oscillation [13]. It is defined by

$$\Delta x=\left|\frac{1}{A}\int_{-\pi/2\omega}^{\pi/2\omega}\int_0^{2\pi}\int_0^a v(r,t)\,r\,dr\,d\theta\,dt\right|, \qquad (12)$$

where $A = \pi a^2$ is the cross-sectional area of the tube. Once the explicit form of the velocity profile is introduced into Eq. (12) and the integration is carried out, we get

$$\Delta x=\left|\frac{2iP_x}{\omega}\left(\frac{1+i\omega t_m}{\beta_v^2\eta}\right)\left(1-\frac{2}{\beta_v a}\frac{J_1(\beta_v a)}{J_0(\beta_v a)}\right)\right| =\frac{2a^2}{\eta\omega}\frac{P_x}{Wo^2}\left|\left(1-\frac{2}{\beta_v a}\frac{J_1(\beta_v a)}{J_0(\beta_v a)}\right)\right|, \qquad (13)$$

where J_1 is the Bessel function of the first kind and first order. As pointed out by Kurzweg [13], in order to avoid direct convective mass transfer, the value of Δx is always taken as smaller than the distance between the fluid reservoirs at different temperature.

2.2. Temperature profile

The corresponding fluid temperature $T(r,x,t)$ within the tube is described by the heat transfer equation (3) which in cylindrical coordinates and under the previous assumptions is expressed as

$$\frac{\partial T}{\partial t}+v\frac{\partial T}{\partial x}=\alpha\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial x^2}\right). \qquad (14)$$

If we consider that the tube wall is thermally insulated and that the fluid temperature cannot diverge at the origin, Eq. (14) must satisfy the boundary conditions

$$\frac{\partial T}{\partial r}(a,t)=0, \qquad (15)$$

$$T(0,t)=\text{finite}. \qquad (16)$$

To get the analytical solution of Eq. (14) with conditions (15) and (16) for the velocity profile under consideration is not a trivial task. In fact, Kurzweg addressed the problem through an approximate

[1] Note that the elastic behavior may be manifested when $De > 1$ which for instance can occur, for a given relaxation time, if the characteristic length scale a is sufficiently small. Also, elastic behavior may appear when the oscillating frequency is very large such that $W_i = \omega t_m \gg 1$.

solution valid only for small values of the product $Pr\,Wo^2$ [12]. Note that in a motionless fluid where only pure molecular diffusion of heat exists, the axial temperature gradient is constant. Following Kurzweg [13], we note that in the geometry considered here, the time-averaged axial temperature gradient, $\gamma = \partial T/\partial x$, is also constant. Due to this fact, we propose a solution given as the real part of the expression

$$T(r,x,t) = \gamma[x + g(r)e^{-i\omega t}], \tag{17}$$

which, incidentally, reproduces the constant time-averaged axial temperature gradient while accounts for the time-dependent cross-stream variation of the temperature through the term $g(r)e^{-i\omega t}$. Substituting Eq. (17) into Eq. (14) yields

$$\frac{d^2 g(r)}{dr^2} + \frac{1}{r}\frac{dg(r)}{dr} + i\frac{\omega}{\alpha}g(r) = \frac{V(r)}{\alpha}, \tag{18}$$

which is a non-homogeneous zeroth-order Bessel equation. The general solution of Eq. (18) is

$$g(r) = c_1 J_0(\beta_T r) + c_2 Y_0(\beta_T r) + g_p(r), \tag{19}$$

where $\beta_T^2 \equiv i\omega t_m Pr/a^2 De = i\,Pr\,Wo^2/a^2$, $g_p(r)$ is a particular integral of Eq. (18) and Y_0 is the Bessel function of the second kind and zeroth order. The function $g_p(r)$ is obtained by the method of undetermined coefficients, as indicated in the appendix. From condition (16), we have $g(0)$ = finite, and to avoid the divergence of Y_0 at the origin we set $c_2 = 0$. Therefore, the general solution can be cast into the form

$$g = c_1 J_0(\beta_T r) + \frac{\Phi(\omega)P_x}{a\alpha(\beta_v^2 - \beta_T^2)}\frac{J_0(\beta_v r)}{J_0(\beta_v a)} + \frac{\Phi(\omega)P_x}{a\alpha\beta_T^2}. \tag{20}$$

Due to the thermally insulated condition (15), the function $g(r)$ must satisfy $g'(a) = 0$. Hence, we finally arrive at

$$g(r) = \frac{\Phi(\omega)P_x}{a\alpha(\beta_v^2 - \beta_T^2)}\left[-\frac{\beta_v}{\beta_T}\frac{J_1(\beta_v a)}{J_0(\beta_v a)}\frac{J_0(\beta_T r)}{J_1(\beta_T a)} + \frac{J_0(\beta_v r)}{J_0(\beta_v a)} + \frac{(\beta_v^2 - \beta_T^2)}{\beta_T^2}\right]. \tag{21}$$

With Eq. (21) the temperature distribution of the fluid inside the tube is completely determined from Eq. (17). Two important points must be stressed at this stage. Firstly, we have found an exact locally valid solution of Eq. (14), in contrast with the approximate solution obtained by Kurzweg [12]. Secondly, our solution is valid for both Maxwell and Newtonian fluids in the appropriate limit. In fact, as stated before, the Newtonian limit is obtained by taking $t_m \to 0$ or, equivalently, $De \to 0$.

3. Enhanced thermal diffusivity

In order to calculate the enhanced heat transfer that takes place between the hot and cold extremes of the tube, we calculate the effective averaged thermal diffusivity, α_e, which is based on the velocity and temperature distribution of the fluid within the tube, obtained in the previous section. Similarly to Kurzweg [13], we neglect the small contribution due to axial thermal conduction, so that the effective averaged thermal diffusivity can be defined as

$$\alpha_e\gamma = -\frac{\omega}{2\pi a^2}\int_0^{\frac{2\pi}{\omega}}\int_0^a [T(r,x,t)]_R[v(r,t)]_R\, r\,dr\,dt, \tag{22}$$

where the subscript R denotes the real part of the corresponding variable. The left-hand side of Eq. (22) represents the effective axial thermal flux per unit cross-sectional area and the right-hand side, the time-averaged convective thermal flux produced by the interaction of the cross-stream-varying velocity and temperature profiles.

Then, substituting the explicit forms for T and v into (22) and performing the time integration leads to

$$\alpha_e = -\frac{1}{2a^2}\int_0^a [\overline{V(r)}g(r) + \overline{g(r)}V(r)]r\,dr, \tag{23}$$

where the overbar denotes complex conjugation. The effective thermal diffusivity can be conveniently normalized by the quantity $\omega(\Delta x)^2$, given in terms of the tidal displacement (13). Explicitly, we have

$$\omega(\Delta x)^2 = \left|\frac{4P_x^2 a^6}{\alpha\eta^2 Pr Wo^6}\left[1 - \frac{2J_1(\beta_v a)}{\beta_v a J_0(\beta_v a)}\right]^2\right|. \tag{24}$$

Introducing the expressions for $V(r)$ and $g(r)$ [cf. Eqs. (9) and (21)] into Eq. (23) and carrying out the radial integration, yields

$$\begin{aligned}\frac{\alpha_e}{\omega(\Delta x)^2} = {} & \frac{Pr}{8a^3}\frac{Wo^2}{\left|\left[1 - \frac{2}{\beta_v a}\frac{J_1(\beta_v a)}{J_0(\beta_v a)}\right]^2\right|}\left\{\frac{1}{(\bar\beta_T^2 - \bar\beta_v^2)}\frac{J_1(\bar\beta_v a)}{J_0(\bar\beta_v a)}\right.\\ & \times\left[\frac{\bar\beta_v}{(\bar\beta_T^2 - \beta_v^2)} - \frac{\bar\beta_v\beta_v}{\bar\beta_T(\bar\beta_T^2 - \beta_v^2)}\frac{J_0(\bar\beta_T a)}{J_1(\bar\beta_T a)}\frac{J_1(\beta_v a)}{J_0(\beta_v a)}\right.\\ & \left.+\frac{\bar\beta_v}{\beta_v^2 - \bar\beta_v^2} + \frac{\bar\beta_v(\bar\beta_T^2 - \bar\beta_v^2)}{(\bar\beta_v^2 - \beta_v^2)(\beta_v^2 - \beta_T^2)}\right] + \frac{1}{(\beta_T^2 - \beta_v^2)}\frac{J_1(\beta_v a)}{J_0(\beta_v a)}\\ & \times\left[\frac{\beta_v}{(\beta_T^2 - \bar\beta_v^2)} - \frac{\beta_v\bar\beta_v}{\beta_T(\beta_T^2 - \bar\beta_v^2)}\frac{J_0(\beta_T a)}{J_1(\beta_T a)}\frac{J_1(\bar\beta_v a)}{J_0(\bar\beta_v a)}\right.\\ & \left.\left.+\frac{\beta_v}{\bar\beta_v^2 - \beta_v^2} + \frac{\beta_v(\beta_T^2 - \beta_v^2)}{(\beta_v^2 - \bar\beta_v^2)(\bar\beta_v^2 - \bar\beta_T^2)}\right]\right\}.\end{aligned} \tag{25}$$

We remark that Eq. (25) is the main result of this paper. For the numerical evaluation of this equation we use the software package *Mathematica* [25]. First, we verify that in the appropriate limit, Eq. (25) can recover Kurzweg's results for the heat transfer enhancement with a Newtonian oscillating fluid. Since the effective thermal diffusivity for an oscillating flow in a tube found by Kurzweg is based on the approximation $Pr\,Wo^2 \ll 1$ [12], a full comparison with his results in that case is not possible. Instead, a comparison is made against the results presented by Kurzweg for the case of an oscillating Newtonian viscous flow within a parallel-plate channel [13] where the quantity $\alpha_e/\omega(\Delta x)^2$ versus the Womersley number is plotted for different Prandtl numbers. A similar plot arising from the use of Eq. (25) in the limit $De \to 0$ is displayed in Fig. 1 for Pr = 1, 10 and 1000. This plot clearly shows that for each Prandtl

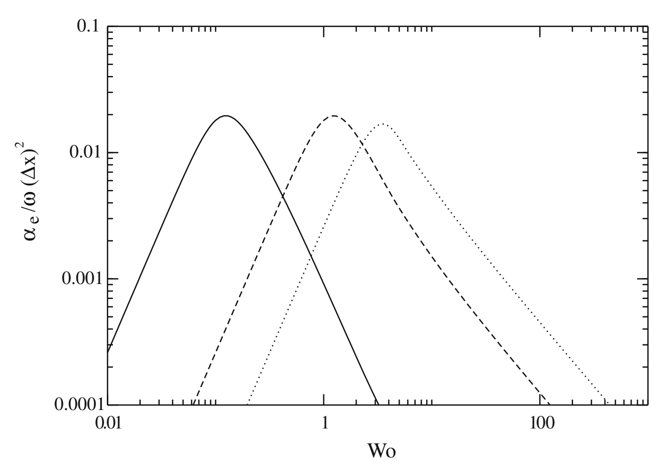

Fig. 1. Effective thermal diffusivity, α_e, normalized by $\omega(\Delta x)^2$ in the Newtonian limit ($De = 0$) as a function of the Womersley number, Wo, for different Prandtl numbers: Pr = 1000 (solid line); Pr = 10 (dashed line); Pr = 1 (dotted line).

number, a maximum in $\alpha_e/\omega(\Delta x)^2$ is found for a given *Wo* in agreement with the results of Ref. [13] for a parallel-plate channel. Note that, in spite of the geometrical differences, the values of the maxima in our case ($\approx$0.02) are very close to those of Kurzweg [13]. The present result confirms that the enhanced thermal diffusivity is produced by the interaction between the velocity profile and the temperature distribution inside the tube.

The presence of elastic effects in the fluid, as reflected by a nonzero Deborah number, has indeed an important influence on the enhanced thermal diffusivity, which now displays resonant behavior. This is illustrated in Figs. 2 and 3. In the first one, taking a fixed $Pr = 10$, the presence of a second maximum in the enhanced thermal diffusivity (within the same frequency interval considered in Fig. 1) is shown for $De = 0.01$, $De = 0.1$ and $De = 1$. This second maximum shifts to lower frequencies and its magnitude grows as the Deborah number is increased; at $De = 1$ it is of the same magnitude as the one of the purely Newtonian fluid. For even higher Deborah numbers and also with a fixed Prandtl number (cf. Fig. 3) further maxima appear in the same frequency interval and the effective thermal diffusivity for the Maxwellian fluid may be orders of magnitude higher than the one of the Newtonian case. Note also that the lowest Deborah number considered in Fig. 2, namely $De = 0.01$, is much lower than the one ($De = 0.085$) for which the dynamic permeability manifests the change from Newtonian to viscoelastic behavior [6]. It has to be emphasized that the existence of multiple maxima in the effective thermal diffusivity for the viscoleastic case, in contrast to the single maximum found for the Newtonian fluid, is directly related to the nature of the Maxwell fluid. In other words, the existence of a resonant behavior manifests the interaction between the viscous dissipative effects and the elastic properties of the material. In fact, a resonant behavior has also been found theoretically and experimentally in studies of the dynamic permeability of viscoealstic fluids [7,9,10].

It is also important to mention that, for Deborah numbers greater or equal than one, the position of the different maxima as a function of *Wo* appearing in the enhanced thermal diffusivity does not shift with varying Prandtl number, as can be seen in Fig. 4 where we have chosen $De = 10$. This feature differs from the Newtonian case, shown in Fig. 1, in which the maxima occur at different *Wo* numbers for each Prandtl number. Note also that in the Newtonian fluid the maxima have approximately the same magnitude. In contrast, in the viscoelastic case the first maximum occurs at the same *Wo* for different Prandtl numbers, but the corresponding maxima now have different magnitudes. As pointed out before, there may be a quite remarkable enhancement in the effective thermal diffusivity when a viscoelastic fluid is used. This could lead to an interesting heat pumping process with technological applications. In particular the possibility arises of applying the previous result to nanofluids with suspended particles [2,26], in instances where such fluids may be described as viscoelastic fluids.

From the practical point of view, it is also important to have a quantitative estimation of the heat transfer enhancement. With this aim, let us compare the axial heat flux under oscillatory conditions, q''_o, with the purely molecular heat flux in the same direction, q''_m. For a given, temperature gradient, we have that $q''_o = -\rho c\alpha_e(\partial T/\partial x)$ and $q''_m = -k(\partial T/\partial x)$. Therefore, the ratio of the heat fluxes q''_o and q''_m becomes

$$H_x = \frac{-k\hat{\alpha}_e \frac{\partial T}{\partial x}}{-k\frac{\partial T}{\partial x}} = \hat{\alpha}_e, \qquad (26)$$

where $\hat{\alpha}_e = \alpha_e/\alpha$ is the dimensionless effective thermal diffusivity. This result shows that for a fluid with constant properties, the heat

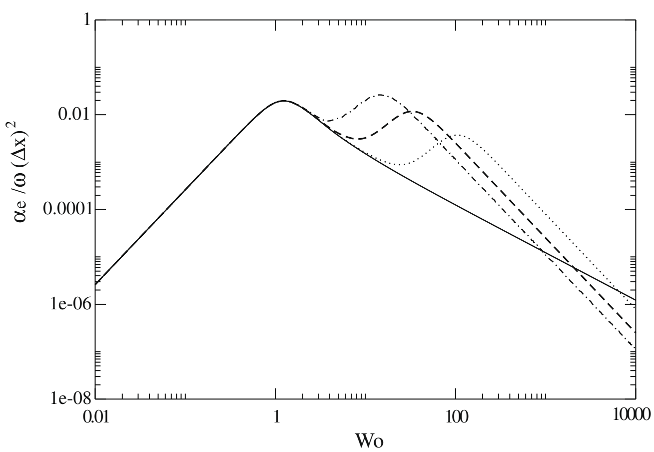

Fig. 2. Normalized effective thermal diffusivity as a function of the Womersley number, *Wo*, for a fixed Prandtl number, $Pr = 10$, and various Deborah numbers. Newtonian fluid: $De = 0$ (solid line); Maxwell fluid: $De = 0.01$ (dotted line); $De = 0.1$ (dashed line); $De = 1$ (dot-dashed line).

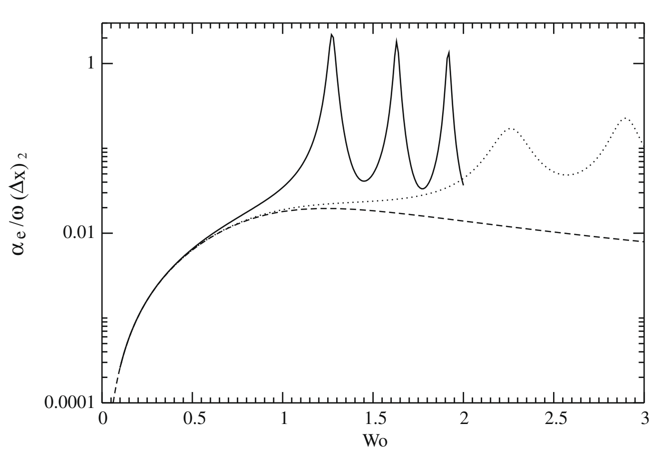

Fig. 3. Normalized effective thermal diffusivity as a function of the Womersley number, *Wo*, for a fixed Prandtl number, $Pr = 10$, and various Deborah numbers. Newtonian fluid: $De = 0$ (dashed line); Maxwell fluid: $De = 1$ (dotted line); $De = 10$ (solid line). In this last case the curve has not been drawn beyond $Wo = 2$ because the number and closeness of the resulting further peaks would only mess up the figure without providing any extra information.

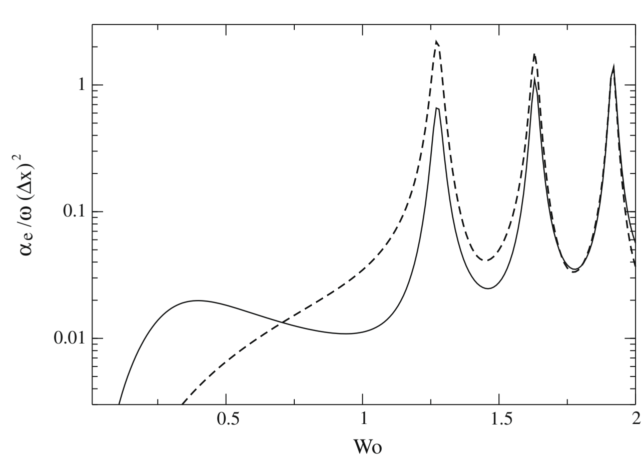

Fig. 4. Normalized effective thermal diffusivity as a function of the Womersley number, *Wo*, for a fixed Deborah number, $De = 10$ and two Prandtl numbers: $Pr = 10$ (dashed line); $Pr = 1000$ (continuous line). Clearly the resonant frequencies in this case do not depend on *Pr*.

flux under oscillatory motion is $\hat{\alpha}_e$ times the molecular heat flux. Therefore, for values $\hat{\alpha}_e > 1$, the oscillatory motion of the fluid leads to an effective heat transfer enhancement.

Let us now consider specific examples of the enhanced thermal diffusivity. First, we take a viscoelastic fluid that has been used to analyze experimentally the dynamic response of oscillatory flows, where a resonant behavior of the dynamic permeability has been observed [9,10]. The fluid is an aqueous solution of cetylpyridinium chloride and sodium salicylate (CPyCl/NaSal) which is known to exhibit the rheological behavior of a linear Maxwell fluid in a range of concentrations [27]. With a 40:40 concentration, the known properties of the fluid at 25 °C are the following: η = 30 Pa s, ρ = 1005 kg/m^3, t_m = 1.25 s [10]. In the experiments, the fluid was set in oscillation by the harmonic motion of a piston within a tube of 0.5 m length and a radius a = 0.025 m [9,10]. Under these conditions, the Deborah number is De = 59.7. The amplitude of the pressure gradient can be calculated as $P_x = \rho x_o \omega^2$, where x_o = 0.01 m is the displacement amplitude of the piston. The thermal diffusivity of the fluid is not available. Since it is an aqueous solution with a viscosity much higher than the viscosity of water, we will consider two Prandtl numbers one and two orders of magnitude higher than that of water, namely, Pr = 100 and Pr = 1000. Heat transfer enhancement requires a tuning process. The plots of the normalized effective thermal diffusivity versus the Womersley number are used to estimate the value the enhanced diffusivity under resonant conditions. For the given De and Pr, the Womersley number for resonant conditions is determined and, therefore, the resonant frequency is also obtained. Hence, in terms of the tidal displacement and the resonant oscillation frequency, the enhanced thermal diffusivity is given by $\alpha_e = n_i\omega(\Delta x)^2$, where n_i is the value of the local maximum of the ratio $\alpha_e/\omega(\Delta x)^2$ for the ith resonant Womersley number. The results obtained for the two Prandtl numbers are shown in Table 1. For Pr = 100, the first resonant Womersley number leads to an effective enhancement, namely, $\hat{\alpha}_e = 9$. In turn, for Pr = 1000 the first resonance gives no enhancement, but the second resonance leads to $\hat{\alpha}_e = 12$. For the conditions and high Prandtl numbers considered, the estimated temperature difference produced by viscous heat generation is negligible ($\Delta T \approx 10^{-3}$–10^{-4} K).

The previous example corresponds to a macroscopic application. We now address a nanomechanical resonator operating at high frequencies. Karabacack et al. [17] have studied these systems in a gaseous environment while Verbridge et al. [28] analyzed different liquids, including water. Here we consider a nano channel filled with water with a radius of a = 100 nm and a length L = 1 μm. Similar dimensions have been considered in recent experimental studies [28]. The physical properties of water at 25 °C are ρ = 996 kg/m^3, $\eta = 8.68 \times 10^{-4}$ kg/m s, $\alpha = 1.5 \times 10^{-7}$ m^2/s, so that Pr = 5.77. We assume that the pressure change during oscillations is of the order of two atmospheres (2×10^5 Pa) which is within the order of magnitude used in nanomechanical resonators [17]. With the considered length, the amplitude of the pressure gradient is $P_x = 2 \times 10^{11}$ Pa/m. In gases, for instance nitrogen, the relaxation time at nanoscales ranges from nanoseconds to microseconds and, according to the work pressure, seems to follow the relation $t_m \propto 1/p$ [17]. For liquids, the relaxation time is expected to be higher than for gases. Unfortunately, detailed information of the relaxation time of water at nanoscales is not available. Therefore, in order to estimate the effective thermal diffusivity in water nano channels, we have considered different relaxation times that vary from 20 ns to 2×10^4 ns. The lower limit (t_m = 20 ns) matches the relaxation time of nitrogen at 1 atm [17]. Results are shown in Table 2 where the relaxation time and the corresponding Deborah number appear in the first and second columns. The third and fourth columns show the Womersley numbers of the first resonance and the values of the maxima, while the fifth column shows the corresponding resonant frequencies. Notice that all resonant frequencies are in the order of MHz, as those used in water nanoresonators [28]. Finally, the dimensionless effective thermal diffusivities shown in the sixth column, predict a heat transfer enhancement except for the lower relaxation time.

Table 1
Parameters at resonant conditions of the effective thermal diffusivity for a CPyCl/NaSal solution oscillating in a tube of radius a = 0.025 m, considering different Prandtl numbers. The corresponding Deborah number is 59.7.

Pr	Wo	n_i	$\omega/2\pi$ (Hz)	P_x (Pa/m)	Δx (m)	$\hat{\alpha}_e$
10^2	0.815 (1st)	8 (1st)	5 (1st)	10,114	3×10^{-3}	9
10^3	0.13 (1st)	0.02 (1st)	0.12 (1st)	5.6	5×10^{-5}	3×10^{-6}
10^3	0.815 (2nd)	1.1 (2nd)	5 (2nd)	10,114	3×10^{-3}	12

Table 2
Parameters at resonant conditions of the effective thermal diffusivity for water oscillating in a nano tube of radius a = 100 nm considering different relaxation times.

t_m (ns)	De	Wo	n_1	$\omega/2\pi$ (MHz)	$\hat{\alpha}_e$
20	1.74	1.95	0.4	52.7	0.004
200	17.4	1.15	2.7	18.3	18.7
2000	174.3	0.62	10.2	5.3	8
20,000	1743	0.35	35	1.7	42

4. Conclusions

In this paper, we have analyzed the heat transfer enhancement in an oscillatory flow of a viscoelastic (Maxwell) fluid in tubes. Through analytical solutions for the velocity and temperature distributions of the fluid inside the tube valid for both Maxwellian and Newtonian fluids, we have obtained an expression of the effective thermal diffusivity. In the case where the relaxation time goes to zero (Newtonian limit) our result agrees with that of Kurzweg [13]. As a matter of fact, within the oscillatory laminar flow conditions, we have extended the work of Kurzweg by providing an analytic result for *any* value of $Wo^2 Pr$ and also by accounting for the viscoelastic properties of the Maxwell fluid. The inclusion of the elastic properties of the fluid leads to interesting features not present in the Newtonian case. In particular, for several specific resonant frequencies, a dramatic enhancement in the effective thermal diffusivity may occur when a viscoelastic fluid is used. In this instance, apart from the dependence on the Womersley and Prandtl numbers, we have a dependence on the Deborah number. In fact, the maximum value of $\alpha_e/(\omega \Delta x^2)$ as a function of Wo may be greater than the one for the Newtonian case for the same Prandtl number. We find that the larger the Deborah number the larger the value of the maximum. We provided a specific example of heat transfer enhancement using a standard viscoelastic fluid (CPyCl/NaSal) in a device with a scale of centimeters. Since the Deborah number increases as the characteristic length decreases, a possible application of this result may be in heat transport under nanofluidic conditions or flow in capillary tubes. In order to illustrate this possibility, considering similar conditions as those used in water nano resonators [28], we have predicted an effective heat transfer enhancement using water under oscillatory conditions at nano scales.

Given the recent interest in high-frequency oscillatory flows for micro- or nano-scale applications in which rheological behavior is manifested, we hope that the consequences of considering a non-Newtonian fluid for heat transfer as derived in this paper may provide new insights for the use of oscillatory flows at small scales.

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Appendix A. Derivation of g_p

We assume that $g_p(r)$ is given by

$$g_p(r) = \delta_1 J_0(\beta_v r) + \delta_2, \tag{A.1}$$

where δ_1 and δ_2 need to be determined. After substitution of Eqs. (A.1) and (9) into Eq. (18) one gets

$$\delta_1 J_0''(\beta_v r) + \frac{1}{r}\delta_1 J_0'(\beta_v r) + i\frac{\omega}{\alpha}[\delta_1 J_0(\beta_v r) + \delta_2] = \frac{\Phi(\omega)}{a\alpha}\left[1 - \frac{J_0(\beta_v r)}{J_0(\beta_v a)}\right]P_x. \tag{A.2}$$

Use of the Bessel function identities allows us to write

$$-\delta_1 \beta_v^2 J_0(\beta_v r) + \delta_1 i\frac{\omega}{\alpha}J_0(\beta_v r) + \delta_2 i\frac{\omega}{\alpha} = \frac{\Phi(\omega)}{a\alpha}\left[1 - \frac{J_0(\beta_v r)}{J_0(\beta_v a)}\right]P_x, \tag{A.3}$$

which after simplification becomes

$$-\delta_1\left(\beta_v^2 - i\frac{\omega}{\alpha}\right)J_0(\beta_v r) + \delta_2 i\frac{\omega}{\alpha} = \frac{\Phi(\omega)P_x}{a\alpha} - \frac{\Phi(\omega)P_x}{a\alpha}\frac{J_0(\beta_v r)}{J_0(\beta_v a)}. \tag{A.4}$$

Therefore, it follows that the undetermined coefficients are given by

$$\delta_1 = \frac{\Phi(\omega)P_x}{a\alpha(\beta_v^2 - \beta_T^2)J_0(\beta_v a)} \quad \text{and} \quad \delta_2 = \frac{\Phi(\omega)P_x}{a\alpha\beta_T^2}. \tag{A.5}$$

Hence, the particular integral g_p is given by

$$g_p = \frac{\Phi(\omega)P_x}{a\alpha(\beta_v^2 - \beta_T^2)}\frac{J_0(\beta_v r)}{J_0(\beta_v a)} + \frac{\Phi(\omega)P_x}{a\alpha\beta_T^2}. \tag{A.6}$$

Substitution of this result into Eq. (19) yields Eq. (20) of the text.

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